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MODULATION AND DETECTION OF OPTICAL SIGNALS

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Notes for Lecture XI, Course X449, Detection of Infrared Radiation --
Heterodyning in the IR

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I. INTRODUCTION

This paper provides a summary of information related to the modulation and detection of information on optical carriers. It is not intended to be a thorough investigation of either modulation or detection, but rather to supplement other more detailed works by emphasizing the treatment of information transfer through an entire system. The summary looks at the most common configurations; intensity modulation, amplitude modulation, frequency or phase modulation, and both direct and coherent detection. In assessing these configurations information capacity and message signal-to-noise ratio are used as a basis of comparison.

The physical and geometric treatment of optical heterodyne (or coherent) detection is given in some detail, since this is the principal topic for the lecture. The advantages of coherent detection in the infrared are evident because the lack of intrinsic gain in infrared photodetectors makes thermal noise insurmountable. Now that coherent detection techniques are available, an enormous improvement in detector sensitivity is possible. To illustrate this point, two communication system concepts are compared, one using direct detection and a photomultiplier detector at $1.06 \mu\text{m}$, and the other using coherent detection with an infrared photodiode at $10.6 \mu\text{m}$.

II MODULATION OF OPTICAL CARRIERS

Modulation of an optical carrier differs from modulation of a radio frequency carrier primarily because of the characteristics and limitations of the devices used for performing the optical modulation. At optical frequencies many modulators are designed to operate directly upon the carrier intensity (amplitude squared of the electric field) rather than the amplitude of the carrier as is common with radio frequency modulators. It is convenient that in direct (quantum) detection of an optical signal, the amplitude of the detector current is proportional to the carrier intensity. For this reason, amplitude modulation of the carrier is of little interest in direct detection schemes. Further, its production is achievable only for extremely small indices and is useful only for special purposes. However, phase modulation, frequency modulation and polarization modulation are easily achieved in the optical spectrum.

Electro-optic modulators obey the relation

$$P(t) = P_0 \sin^2 \Gamma(t) \sin^2 \omega_c t$$

where $\Gamma(t)$ is the retardation introduced by the modulating voltage = $\Gamma_0 + \Gamma_m \sin \omega_m t$

ω_c is the optical carrier frequency

P_0 is the peak power of the laser carrier before entering the modulator.

In order to be able to analyze this modulation by conventional means, we begin by expanding the modulating term

$$\begin{aligned} \sin^2 \Gamma(t) &= \frac{1}{2} [1 - \cos(2\Gamma_0) J_0(2\Gamma_m)] && \text{DC term} \\ &+ \sin 2\Gamma_0 J_1(2\Gamma_m) \sin \omega_m t && \text{fundamental} \\ &- \cos 2\Gamma_0 J_2(2\Gamma_m) \cos 2\omega_m t && \text{2nd harmonic} \\ &+ \sin 2\Gamma_0 J_3(2\Gamma_m) \sin 3\omega_m t && \text{3rd harmonic} \\ &\dots && \text{etc.} \end{aligned}$$

The electro-optic retardation contains the bias term Γ_0 which is adjusted such that with no modulation, half the power is transmitted through the modulator. The modulating term $\sin^2 \Gamma(t)$ can then be approximated by

$$\frac{1}{2} [1 + \Gamma_m \sin \omega_m t].$$

1. Intensity Modulation

In optical systems where an electro-optic modulator is used external to the laser source the output may be either in the form of loss modulation at a fixed polarization or it may be polarization modulated. The basic form for loss (intensity) modulation is

$$P(t) = P_0/2 [1 + mM(t)] \sin^2 \omega_c t \quad (1)$$

where P_0 is the power of the laser carrier before entering the modulator

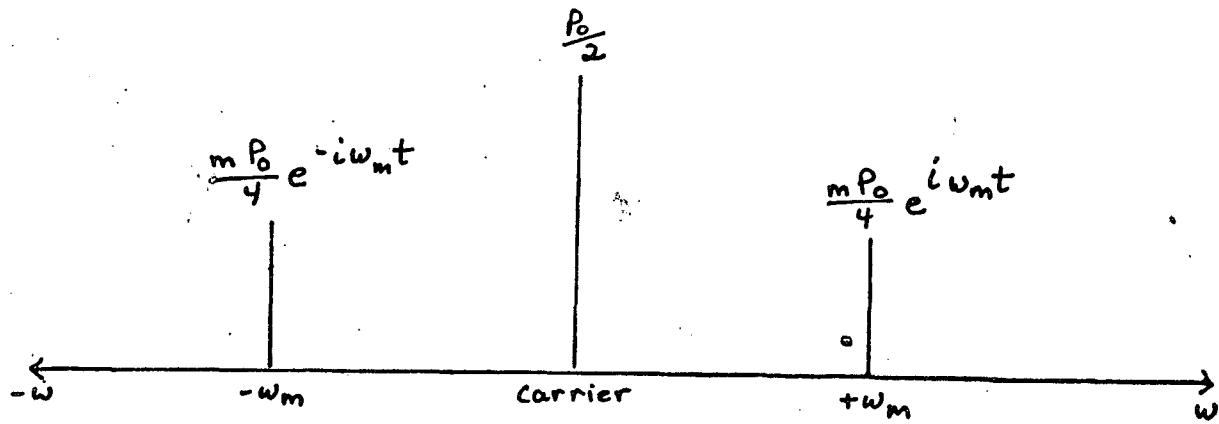
$P_0/2$ is the average laser carrier power out of the modulator (biased power without modulation)

m is the modulation index defined as 1.0 for 100% modulation*

$M(t)$ is the message function, balanced, having maximum values of ± 1.0 . Example: $M(t) = \cos \omega_m t$ where ω_m is the modulating frequency.

It is convenient to examine the exponential form of the modulation in order to examine the various sidebands and their relative intensities. Assuming $M(t)$ is a periodic sinusoidal function,

$$P(t) = P_0/2 [1 + m/2(e^{i\omega_m t} + e^{-i\omega_m t})] \sin^2 \omega_c t.$$



* Note: m is related to the electro-optic peak retardation Γ_m
 $m = 2 \sin (2\Gamma_0) J_1(2\Gamma_m) \approx \Gamma_m - \Gamma_m^3/2 + \Gamma_m^5/12 - \dots$

Note that $\frac{1}{2}$ the power is in the optical carrier and $\frac{1}{2}$ is in the message function.

2. Subcarrier Modulation

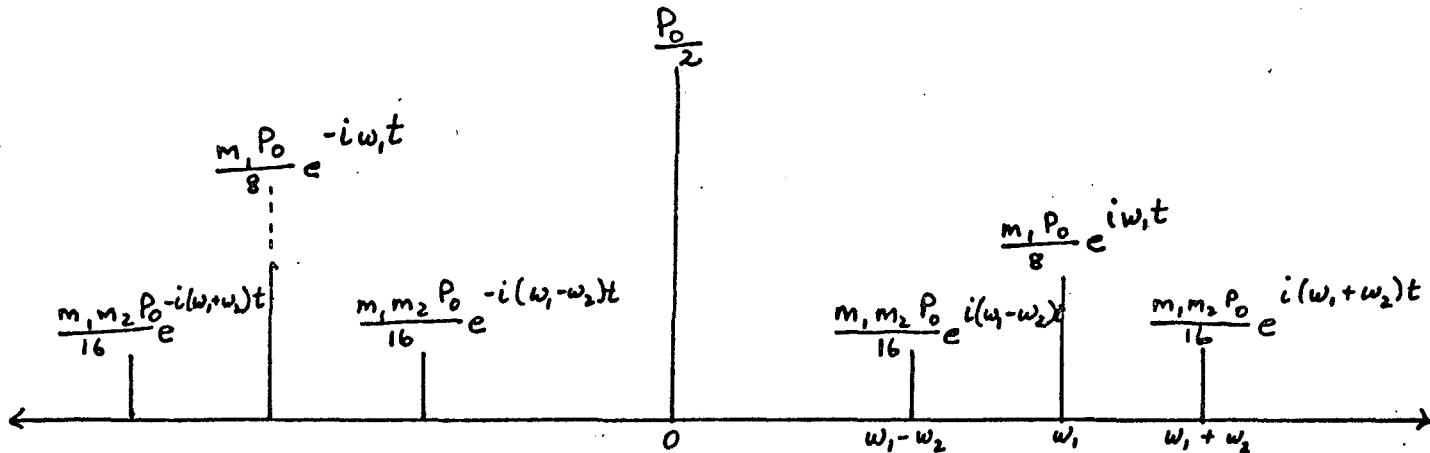
In subcarrier modulation the basic unmodulated carrier appears as a single tone ω_1 intensity modulation.

$$P = P_0/2 [1 + m_1 \cos \omega_1 t] \sin^2 \omega_c t.$$

However, the message function is not contained in the above expression. Instead, the message function is multiplied with the carrier to form the expression

$$P = P_0/2 [1 + \frac{1}{2}(1 + m_2 \cos \omega_2 t) m_1 \cos \omega_1 t] \sin^2 \omega_c t \quad (2)$$

where now m_2 is the modulation index of the message function on the subcarrier and m_1 is the modulation index of the subcarrier on the optical carrier. Assuming a sinusoidal $M(t) = \cos \omega_2 t$, then equation (2) yields the spectrum.



It should be noticed that $\frac{1}{2}$ the power is in the optical carrier term, $\frac{1}{4}$ the power is in the subcarrier term, and $\frac{1}{4}$ the power is in the message function modulation (assuming $m_1 = m_2 = 1$).

3. Polarization Modulation

Polarization modulation is described as modulation of the polarization angle by $\pm 45^\circ$. Two receivers channels which

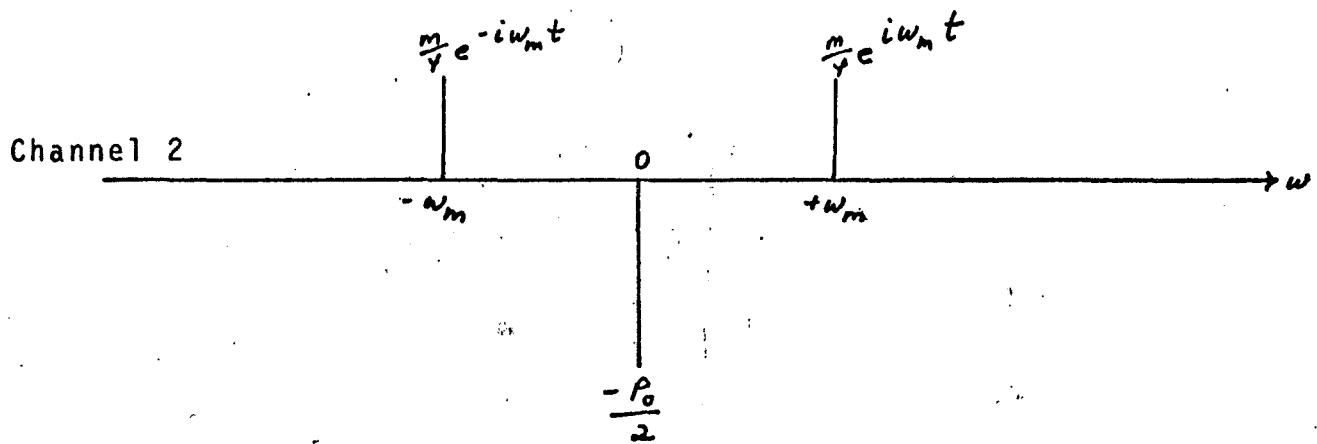
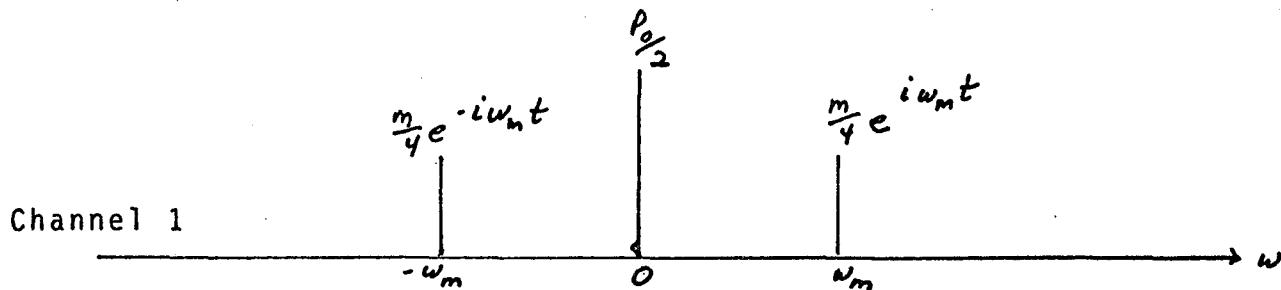
are polarization sensitive are then located at orthogonal polarizations such that half of the power enters each channel. The modulation function is defined by

$$P_1 = P_0/2 [1 + mM(t)] \sin^2 \omega_c t \quad \text{Channel 1} \quad (3)$$

$$P_2 = P_0/2 [1 - mM(t)] \sin^2 \omega_c t \quad \text{Channel 2}$$

P_1 and P_2 may be right and left hand circular polarization or orthogonal linear polarizations. In the detection process, Channel 2 will be subtracted from Channel 1.

The spectrum of the modulation is then identified for each channel as follows:



It will be noticed that the carrier term cancels out and the message terms add such that all the power is used in the transmission of information.

4. Intracavity Frequency Modulation (Optical FM)

Looking at the classical expression for frequency modulated signal, the field is expressed as

$$E(t) = E_0 \cos(\omega t + \beta \sin \omega_m t).$$

The spectrum terms of this signal for large deviation is given by the well known Bessel Function of the first kind

$$E(t) = E_0 \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t.$$

Of practical consideration is the fact that the average power in a phase or frequency modulated wave is constant; the sum of the squares of the individual Fourier components of the modulated wave is equal to unity.

$$P \sim [E(t)]^2 = \frac{E_0^2}{2} \sum_{-\infty}^{\infty} J_n^2(\beta) = \frac{E_0^2}{2}$$

When intracavity optical frequency modulation is used, there are two constraints which affect the resulting modulation spectrum. First, the modulation index is a function of cavity parameters and the peak retardation, but is usually a value less than unity. The low value of FM modulation index differs from the usual rf case where very high values are used. This results in most of the modulated energy being limited to the first order sidebands.

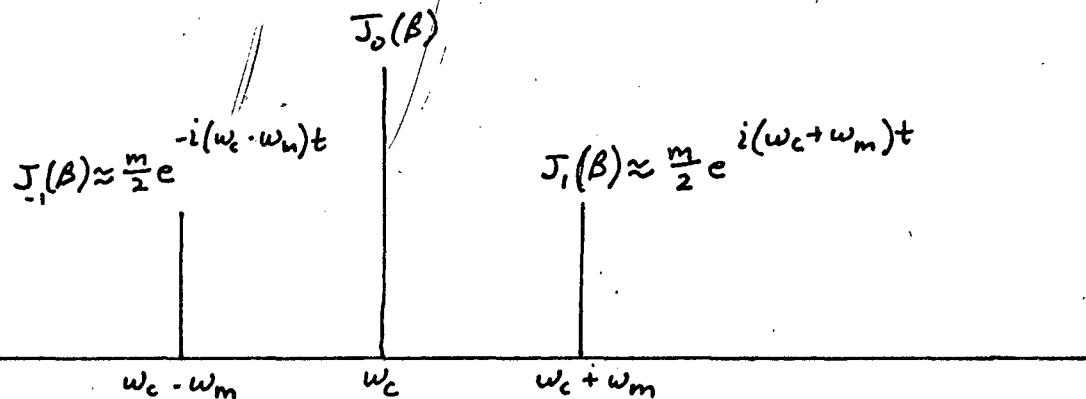
Second, optical frequency modulation is detected with optical heterodyne detection where a conventional i.f. amplifier is used. The optimization of the design of the system calls for an i.f. bandwidth which accommodates only the first order sidebands. Under these circumstances, it is convenient to have an explicit

approximation to the "effective" modulation characteristic.

For modulation index less than 1.4, the approximate characteristic is $3.2\beta^2(1 - 0.55\beta)$, where β is the conventional FM modulation index $\delta f/f_m$.* A hybrid expression may thus be generated which permits the use of an equivalent modulation index $m = 3.2\beta^2(1 - 0.55\beta)$.

$$P \approx P_0 (1 + mM(t)) \sin^2 \omega_c t \quad (4)$$

The spectrum is obtained from the first two terms of the Bessel Series,



In the optimized band-limited situation, m has a maximum value of about 1.5 for $\beta = 1.2$. Under these conditions, the signal power (modulated) is greater than the original carrier power, a condition due to the enhancement effect of frequency modulation.

*NOTE: $\beta = \frac{\delta f}{f_m} = \frac{1}{f_m} \left(\frac{c}{2L} \right) \frac{\Gamma_m}{\pi}$

where c = velocity of light

L = length of laser cavity

Γ_m = peak retardation

f_m = highest modulating frequency

5. Intracavity Coupling Modulation

Coupling modulation is of great interest for infrared lasers because of the enhanced modulation effect and the nearly unlimited bandwidth capabilities of the technique. The basic modulation equation is similar to that for simple intensity modulation.

$$P = P_c \sin^2 \Gamma(t)$$

where P_c is the circulating power in the laser cavity rather than the external output power of the laser. The enhancement offered by the technique is thus equal to the ratio of the circulating power to the optimally coupled output power of the laser. For the carbon dioxide laser at $10.6 \mu\text{m}$, this ratio is about 10 to 1.

The coupling parameter P/P_c , or $mM(t)$, can be expanded as follows:

$$\begin{aligned} mM(t) &= P/P_c = \sin^2 \Gamma(t) \\ &= \frac{1}{2} [1 - \cos 2\Gamma(t)] \end{aligned}$$

$$\text{Let } r(t) = \Gamma_0 + \Gamma_m \sin \omega_m t$$

$$\begin{aligned} mM(t) &= \frac{1}{2} [1 - \cos(2\Gamma_0) J_0(2\Gamma_m t)] && \text{DC term} \\ &\quad + \sin(2\Gamma_0) J_1(2\Gamma_m t) \sin \omega_m t && \text{fundamental} \\ &\quad - \cos(2\Gamma_0) J_2(2\Gamma_m t) \cos 2\omega_m t && \text{2nd harmonic} \\ &\quad + \sin(2\Gamma_0) J_3(2\Gamma_m t) \sin 3\omega_m t && \text{3rd harmonic} \\ &\quad \dots && \text{etc.} \end{aligned}$$

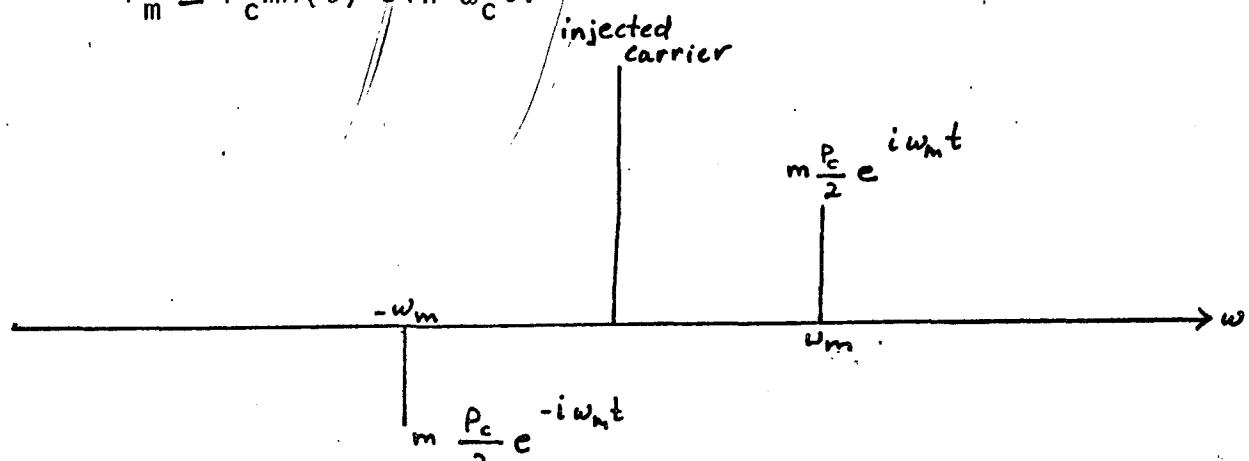
The average power coupled from the laser is proportional to the first term or dc term which is dependent both upon Γ_0 , the dc retardation and Γ_m , the peak ac retardation. The first term may be set to zero which leaves only the even harmonics. This type

of modulation is equivalent to double sideband-suppressed-carrier DSBSC modulation. Injection of a carrier at the receiver restores the fundamental and odd harmonics. The spectral terms which contain information are

$$\begin{aligned}
 mM(t) &= +J_1(2\Gamma_m) \sin \omega_m t \\
 &\quad -J_2(2\Gamma_m) \cos 2\omega_m t + J_3(2\Gamma_m) \cos 3\omega_m t \dots \\
 &= \left(\Gamma_m - \frac{\Gamma_m^3}{3} + \frac{\Gamma_m^5}{12} - \dots \right) \sin \omega_m t - \dots \\
 mM(t) &\approx P_m/P_c = J_1(2\Gamma_m) \sin \omega_m t.
 \end{aligned}$$

Thus, the modulated sideband power is

$$P_m \approx P_c mM(t) \sin^2 \omega_c t. \quad (5)$$



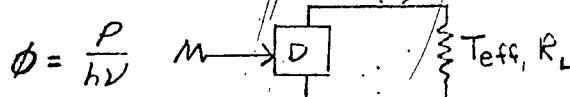
6. Pulse Modulation

Pulse modulation is important for both radar applications and communications where binary information is transmitted in the form of pulse "1" and no-pulse "0". In this form of modulation modulated signal power has no meaning. Since every pulse contains information, there is no energy lost in carriers. The criteria for effective transmission of information over a system of this sort is the probability of detection at the receiver. The spectrum of a single pulse is important only in relation to the information rate, since pulse overlap will result in a reduction in the probability of detection. The parameter of optical energy per pulse has more significance than optical power. Thus, the primary factor in pulse modulation is the single pulse energy E_s :

III DIRECT DETECTION AND DEMODULATION

1. Direct Detection of Carriers

Direct detection is defined as the direct use of electrons produced by ionizations of photons incident on the detector. For example, assume an optical flux of



where P is the optical power and $h\nu$ is the energy per photon.

A signal current is produced in the detector of

$$i_s = \eta G e \frac{P_s}{h\nu}$$

where η is the number of ionizations per photon (quantum efficiency)

G is the number of electrons per ionization (gain)

e is the electronic charge

T_{eff} is the effective temperature of the load resistance

R_L is the load resistance.

The gain of a photomultiplier may be 10^5 or higher, where a single photon can generate thousands of electrons and where individual photon events are easily detected. An avalanche photodiode may have a gain of 10^2 or more. However, most infrared photodiodes do not have intrinsic gain.

The optical power P above may be composed of a signal power P_s and a background power P_b . In such a case, the total current is

$$i = \frac{\eta G e (P_s + P_b)}{h\nu}$$

The power in the resistor is given by

$$i^2 R_L = \frac{\eta^2 G^2 e^2 (P_s + P_b)^2 R_L}{(h\nu)^2}$$

The noise power (shot) in the resistor is the shot noise produced by the direct current in the detector.

$$i_n^2 R_L = 2(Ge)i BRF$$

F is the noise factor defined by the increase in noise introduced by the current gain process. The value of F varies for different types of detectors.

$F = 1$ for photodiodes

$F = 1.3$ for photomultipliers

$F = 2$ for photoconductors

$F > 2$ for avalanche photodiodes.

The load resistor also has thermal noise so that the total noise power in the load resistor is

$$P_n = 2(Ge) i_{dc} BRF + 4kTB.$$

Now i_{dc} in the above equation is the total dc current in the circuit and is the sum of that produced by the signal flux, the background flux, and dark current. In this discussion, we are assuming the dark current to be negligible and that the main shot current is produced by the signal and the background flux

$$i_{dc} = \eta Ge \left(\frac{P_s + P_b}{h\nu} \right).$$

The noise power in the load resistor becomes

$$P_n = \frac{2\eta(Ge)^2(P_s + P_b)BRF}{h\nu} + 4kTB.$$

1.1 Noise-Equivalent-Power (NEP)_T, Thermal Limited

The thermal limited case is that usually encountered with infrared photodiodes and photoconductors used in direct detection processes. Here the signal current generated by the optical signal flux must be greater than the "equivalent thermal current" in the circuit. The thermal noise current i_t in the load resistor is defined in the relation

$$i_t^2 R = 4kTB.$$

The thermal current is a noise current and not a dc current. We now ask the question what equivalent optical noise flux ϕ_t will produce the same amount of noise current, $i_t = \eta Ge\phi_t$. Again, the optical noise flux is a noise modulated flux as opposed to a dc flux. Substituting this expression for i_t in the thermal noise equation above gives

$$i_t^2 R = 4kTB = \eta^2 G^2 e^2 \phi_t^2 R.$$

Solving for the equivalent noise flux

$$\phi_t = \sqrt{\frac{4kTB}{R}} \frac{1}{\eta Ge}$$

Finally, the noise equivalent power $(NEP)_T$ is defined as $h\nu\phi_t$

$$(NEP)_T = \frac{h\nu}{\eta Ge} \sqrt{\frac{4kTB}{R}} \quad (6)$$

1.2 Noise-Equivalent-Power (NEP)_B, Background Limited

For the background limited case, the signal current must be greater than the shot noise produced by the dc background flux. For a background flux of

$$(\phi_b)_{dc} = \frac{P_b}{h\nu}$$

the dc current produced in the detector is

$$i_{dc} = \eta Ge \frac{P_b}{h\nu}$$

Now we define an equivalent noise (ac) background flux which will produce the same shot noise as the above dc current

$$\eta Ge(\phi_b)_{ac} = \sqrt{2e i_{dc} B}$$

where

$$(\phi_b)_{ac} = \frac{(NEP)_B}{h\nu}$$

Solving for $(NEP)_B$

$$(NEP)_B = \frac{h\nu}{\eta Ge} \sqrt{2e i_{dc} B}$$

$$(NEP)_B = \sqrt{2} \left(\frac{h\nu B}{\eta} \right) P_b, \quad \frac{h\nu B}{\eta} = P_g$$

$$(NEP)_B = \sqrt{2} P_g P_b$$

(7)

1.3 Noise-Equivalent-Power (NEP)_S, Signal Shot Noise Limited

The case where signal strength is great or where background and thermal noise is negligible, the noise in the system is determined by signal shot noise. The computation of the signal shot noise is similar to that for background noise where average signal flux is substituted for background flux. The dc current produced in the detector by the average signal current is

$$i_{dc} = \eta Ge \left(\frac{P_s + P_b}{hv} \right)$$

where now as we have said $P_s \gg P_b$.

The signal shot noise power (NEP)_S is thus

$$(NEP)_S = \sqrt{2P_g P_s} \quad (8)$$

1.4 Signal-to-Noise Ratio of Unmodulated Carriers (S/N)_{dc}

In this section we have described the types of noise encountered in direct detection of optical signals. We now define what is meant by the signal-to-noise ratio. We define the (S/N)_{dc} as that for the detection of unmodulated carriers,

$$(S/N)_{dc} = \left(\frac{P_s}{NEP} \right)^2$$

where P_s is the actual optical signal power and NEP is the hypothetical equivalent optical noise power for the three cases described in 1.1 through 1.3. Accordingly, these are

$$\text{Thermal limited } \left(\frac{S}{N} \right)_{dc} = \left[\frac{\eta Ge}{hv} \sqrt{\frac{R}{4kTB}} P_s \right]^2$$

$$\text{Background limited } \left(\frac{S}{N} \right)_{dc} = \left[\frac{P_s}{\sqrt{2P_g P_b}} \right]^2$$

$$\text{Signal Shot limited } \left(\frac{S}{N} \right)_{dc} = \left[\frac{P_s}{\sqrt{2P_g P_s}} \right]^2 = \eta \frac{P_s}{2hvB}$$

$$\text{General } \left(\frac{S}{N} \right)_{dc} = \left[\frac{P_s}{\frac{hv}{\eta Ge} \sqrt{\frac{4kTB}{R}} + \sqrt{2P_g(P_b + P_s)}} \right]^2 \quad (9)$$

The detection of single pulse modulation depends not on the NEP of the detector, but rather the number of photoelectrons necessary for a given detection probability. The $(S/N)_m$ associated with this requirement may be expressed as

$$(S/N)_m = \eta \frac{E_s}{h\nu} = N_s$$

where E_s is the signal energy

N_s is the number of photoelectrons produced per pulse.

Typically, for quantum limited detection and high quantum efficiencies, $N_s \approx 15$ for a probability of error of 10^{-5} .

2. Demodulation and Information Signal-to-Noise (S/N)_m

We have shown in Section II that not all the transmitted optical power contains information and that, in fact, for intensity modulated beams, half the optical power is in the carrier. In computing the information signal-to-noise, therefore, the signal power P_m is that contained in the information sidebands

$$P_m = m \frac{P_s}{2} \text{ and } \left(\frac{S}{N}\right)_m = \left(\frac{P_m}{NEP}\right)^2$$

where P_s is the power of the laser signal without modulator bias. In the following list of expressions, P_s remains as indicative of the available laser signal power available without modulation or modulation bias. The information S/N expressed in this way serves as an excellent means of comparing the relative merit of one modulation and detection technique over another. All examples are signal shot limited.

$$\text{Intensity Modulation } \left(\frac{S}{N}\right)_m = \left[\frac{P_m}{\sqrt{2}P_s} \right]^2 = \eta \frac{m^2 P_s}{8h\nu B} \quad (10)$$

$$\text{Subcarrier Modulation } \left(\frac{S}{N}\right)_m = \eta \frac{m^2 P_s}{16h\nu B} \quad (11)$$

$$\text{Polarization Modulation } \left(\frac{S}{N}\right)_m = \eta \frac{m^2 P_s}{2h\nu B} \quad (12)$$

IV COHERENT DETECTION AND DEMODULATION

Direct detection of optical signals is practical in photoemissive devices such as photomultipliers and avalanche photodiodes where large intrinsic gain permits the generation of large numbers of photoelectrons for each photon incident on the photo-surface. Photoemissive devices, however, roll off about $1 \mu\text{m}$ because the energy of the photoelectrons is inversely proportional to the wavelength of the light and the work function of the photocathode becomes too great a barrier to permit photoemission. From about $1.1 \mu\text{m}$ out to the far infrared, the best optical detectors are semiconductor photodiodes. Direct detection of these photodiodes is usually thermal limited detection.

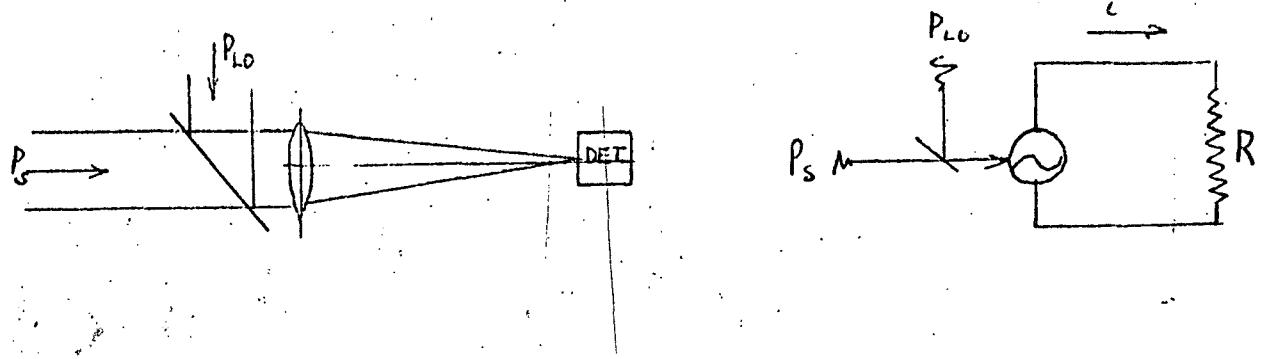
Coherent detection is the process of mixing a local oscillator laser beam with the incoming signal beam. It has the advantages that a conversion gain is achieved through the photoelectric mixing making the detection process quantum limited. In words, it can be described as a method of mixing two fields to produce a current proportional to the product. If one of these fields is the local oscillator field, it can be increased arbitrarily to the point where the shot noise produced is greater than the thermal noise in the circuit.

Coherent detection requires careful alignment between the signal field and the local oscillator field. These geometric considerations become critical at short wavelengths but are generally uncritical at the longer infrared wavelengths. In practice, the $10.6 \mu\text{m}$ wavelength is ideal for the use of optical heterodyne detection, first because the poor performance of direct detection at this wavelength, and second because the longer wavelength makes alignment and phase matching feasible.

1. Physical and Geometrical Considerations

The requirement of phase matching of the wavefronts of the signal and local oscillator beams wavefronts is sufficiently important that inclusion of geometrical considerations with the physical description is necessary. We therefore describe a typical

signal and local oscillator configuration with the geometric layout on the left and the electrical equivalent circuit on the right



By Poynting's theorem, the signal power and the local oscillator power can be written

$$P_s = \frac{1}{2Z_0} \int \vec{E}_s^2 dA = \frac{1}{2Z_0} |E_s|^2 \int dA \quad P_{lo} = \frac{1}{2Z_0} |E_{lo}|^2 \int dA$$

The electric field can be written as the sum of the signal field and the local oscillator field

$$\vec{E} = \vec{E}_s + \vec{E}_{lo}$$

and the current in the detector can be expressed as

$$i = \eta \frac{Ge}{hv} P = \eta \frac{Ge}{hv} \frac{1}{2Z_0} \int (\vec{E}_s^2 + \vec{E}_{lo}^2 + 2\vec{E}_s \cdot \vec{E}_{lo}) dA$$

Both the signal beam and the local oscillator beam are focussed down on the detector surface. Each produces an electrical field distribution on the detector surface which is determined by

$$E_s(r) = \frac{2J_1(kr/F_s)}{kr/F_s} |E_s| \quad E_{lo}(r) = \frac{2J_1(kr/F_{lo})}{kr/F_{lo}} |E_{lo}|$$

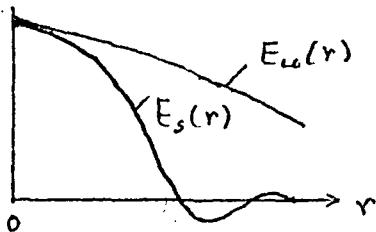
where k is the propagation constant

r is the distance from the center point of the detector

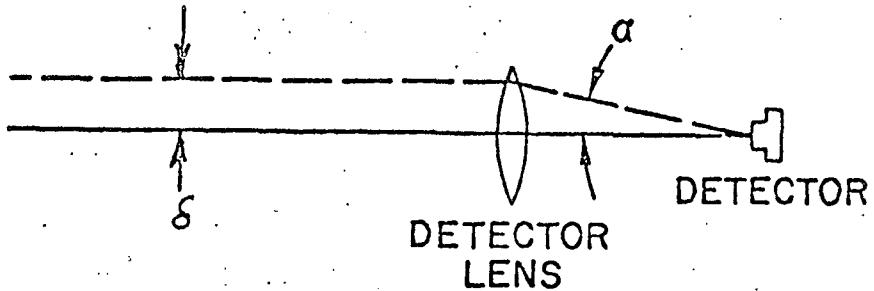
F_s is the F number of the signal beam

F_{lo} is the F number of the local oscillator beam.

The reason for not having equal F numbers for the signal and local oscillator beams is evident when observing these fields displayed graphically. Having a larger F number for the local oscillator makes alignment over the Airy disc of the signal much less difficult.



Another factor which must be taken into consideration is the tilt angle between the signal and local oscillator beams. Tilt can be generated either by misalignment of the incoming beams or by displacement of two parallel beams as shown below.



The tilt angle of α produces a loss of signal current of

$$L(\alpha) = \frac{\sin 2\pi F_s \alpha}{2\pi F_s \alpha}$$

Now the time varying current in the detector at the i.f. frequency is

$$i(t, r) = \eta \frac{G_e}{h\nu Z_0} L(\alpha) \int_0^r E_s(r) E_{L0}(r) 2\pi r dr \cos \omega_{if} t$$

Substituting from Poynting's theorem

$$|E_s|^2 = 2 P_s Z_0, |E_{L0}|^2 = 2 P_{L0} Z_0, |E_s|/|E_{L0}| = 2 \sqrt{P_s P_{L0}} Z_0$$

$$i(t, r) = \eta \frac{G_e}{h\nu Z_0} 2 \sqrt{P_s P_{L0}} \int_0^r \frac{2 J_1(kr/F_s)}{kr/F_s} \cdot \frac{2 J_1(kr/F_{L0})}{kr/F_{L0}} \cdot 2\pi r dr L(\alpha) \cos \omega_{if} t$$

The i.f. signal power is

$$i^2 R = 2 \left(\frac{\eta G e}{h\nu} \right)^2 P_s P_{LO} L(\alpha)^2 \left[\int_0^r \frac{2J_1(kr/F_s)}{kr/F_s} \cdot \frac{2J_1(kr/F_{LO})}{kr/F_{LO}} 2\pi r dr \right]^2 R$$

The shot noise produced by the local oscillator is

$$i_{dc} = \frac{\eta G e}{h\nu} P_{LO} \quad i_r^2 R = 2e i_{dc} BR = 2\eta \frac{e^2 G}{h\nu} BR P_{LO}$$

Finally, the carrier-to-noise ratio can be written as

$$\left(\frac{C}{N} \right)_{if} = \eta \left(\frac{P_s}{h\nu B_{if}} \right) L(\alpha)^2 \left[\int_0^r \frac{2J_1(kr/F_s)}{kr/F_s} \cdot \frac{2J_1(kr/F_{LO})}{kr/F_{LO}} 2\pi r dr \right]^2$$

The geometric terms, $L(\alpha)^2$ and the term containing the integral, are equal to unity if the conditions of phase matching and beam tilt are met. Then the carrier-to-noise ratio in the i.f. becomes

$$\boxed{\left(\frac{C}{N} \right)_{if} = \eta \frac{P_s}{h\nu B_{if}}} \quad (13)$$

2. Required Local Oscillator Power

The condition for quantum limited operation of a heterodyne or homodyne receiver is that the shot noise produced in the detector at the i.f. frequency is sufficient to override the thermal noise in the i.f. amplifier. The shot noise produced by the local oscillator is

$$i_r^2 R = 2e i_{dc} BR = \frac{2\eta e^2 G B R}{h\nu} P_{LO}$$

and the thermal noise in the i.f. amplifier is

$$4kTBN_F = 4kT_{eff} B$$

where N_f is the noise figure of the i.f. amplifier, and T_{eff} is the noise temperature. Then we let

$$\frac{2\eta e^2 GBR}{h\nu} P_{LO} > 4kTB N_f$$

and the required LO power is

$$P_{LO} = \frac{2N_f (kT)(h\nu)}{R} \frac{1}{G^2 \eta} \quad (14)$$

3. Heterodyne Conversion Gain

There are two definitions of heterodyne conversion gain and each should be discussed briefly. The first of these is defined as the ratio of i.f. signal power to optical signal power, a hybrid definition without much meaning

$$\begin{aligned} (\text{Conversion Gain})_1 &= \frac{\text{i.f. signal power}}{\text{optical signal power}} \\ &= \frac{2\left(\frac{\eta G e}{h\nu}\right)^2 P_s P_L R}{P_s} = 2\left(\frac{\eta G e}{h\nu}\right)^2 P_L R \end{aligned}$$

The reason the above definition is meaningless is that an extraordinary conversion loss is encountered in converting optical energy to electrical energy. For example, take any quantum limited detector and let the signal-to-noise ratio equal unity

$$S/N = \eta \frac{P_s}{h\nu B} = 1$$

solving for minimum detectable signal power P_s

$$P_s = \frac{h\nu B}{\eta}$$

Now define conversion loss as the ratio of this signal power to the electrical power out of the detector

$$\text{Loss} = \frac{P_s}{i^2 R} = \frac{\frac{h\nu B}{\eta}}{\frac{(\eta G e)^2 R}{i^2}} = \frac{h\nu B}{\eta^3 G^2 e^2 R}$$

Letting $B = n$, $\lambda = 1 \mu\text{m}$, $R = 100$, $\eta = 0.2$, the conversion loss is

$$\text{Loss} \approx \frac{10^{19}}{G^2 n}$$

Now, even for a photomultiplier where G may be 10^5 , the conversion loss is still $10^9/n$. What this illustrates is that another definition is required for conversion gain.

The second definition is simply the ratio of i.f. signal power to the signal power one would have without heterodyne conversion.

$$\begin{aligned} (\text{Conversion Gain})_2 &= \frac{\text{i.f. signal power}}{\text{direct detection signal power (electrical)}} \\ &= \frac{2 \left(\frac{\eta G e}{h\nu} \right)^2 P_s P_L R}{\left(\eta \frac{P_s}{h\nu} G e \right)^2 R} \end{aligned}$$

$$(\text{Conversion Gain})_2 = \frac{2 P_L}{P_s}$$

(15)

This second definition has a special significance, relating local oscillator power to signal power.

4. Demodulation

Demodulation of coherent carriers has been thoroughly investigated in radio and microwave communications. These techniques apply directly to optical carriers where coherent detection is used. For the present discussion, we will be concerned with three specific cases. The first of these envelope detection of a heterodyne signal where C/N is greater than 10. The second is product detection (homodyne) detection where any C/N applies. The third is for band limited FM detection where only the first sidebands of the modulated signal lie within the i.f. passband of the receiver. The C/N regime of the FM system is arbitrarily chosen to be $C/N > 10$.

Envelope Detection

(Heterodyne)

C/N > 10

$$\left(\frac{S}{N}\right)_m = 2m^2 \left(\frac{C}{N}\right)_{i,f} = n \frac{m^2 P_s}{h\nu B}$$

Product Detection

(Homodyne)

Any C/N

$$\left(\frac{S}{N}\right)_m = 2m^2 \left(\frac{C}{N}\right)_{i,f} = 2n \frac{m^2 P_s}{h\nu B}$$

FM Detection

(Heterodyne)

C/N > 10

$$\left(\frac{S}{N}\right)_m = 3\beta^2(1-.55\beta) \left(\frac{C}{N}\right)_{i,f} = \frac{3\beta^2(1-.55\beta)n P_s}{2h\nu B}$$

B in the above expressions is information bandwidth.

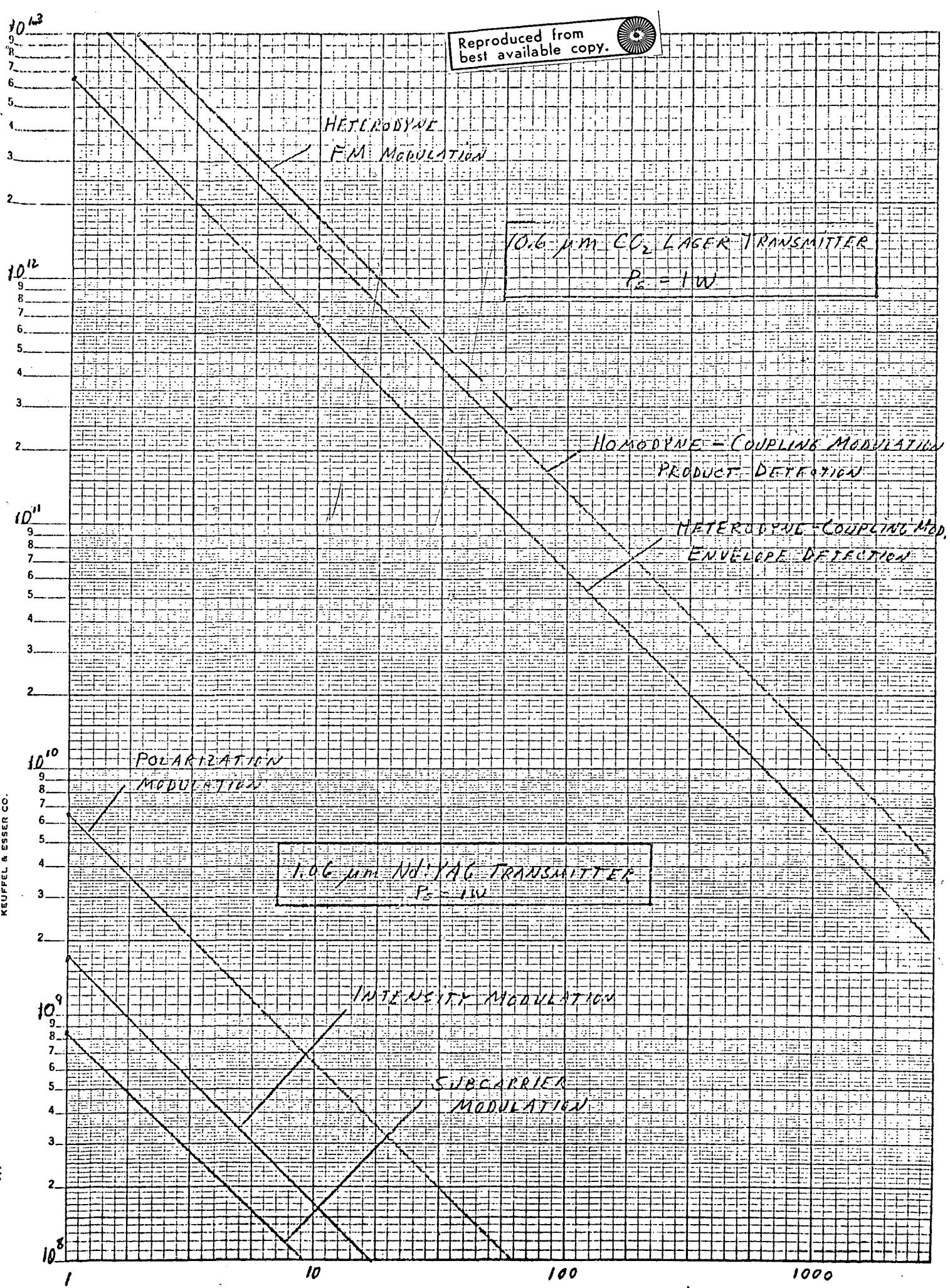
V COMPARISON OF TWO COMMUNICATION SYSTEM CONCEPTS

This section addresses two important system concepts in an attempt to compare their relative performance. One is the Nd:YAG 1.06 μm laser transmitter using intensity modulation, polarization modulation and subcarrier modulation, and using direct quantum limited detection. The other system is the carbon dioxide laser transmitter at 10.6 μm using optical frequency modulation, and coupling modulation. The CO_2 system uses coherent detection with envelope detection, frequency discriminator detection, and phase-lock, or homodyne detection.

One watt of transmitter power is assumed for both systems. The modulation index of 0.5 is assumed for intensity modulation, polarization modulation and coupling modulation; and an index of 1.4 for band-limited optical FM modulation.

A quantum efficiency of 0.01 is assumed for the 1.06 μm detector whereas a quantum efficiency of 0.5 is assumed for the 10.6 μm heterodyne detector.

The attached figure shows the relative performance of these system concepts.



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